Quark masses from lattice QCD and the study of textures

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Abstract

I review how the determination of quark masses from lattice QCD can be used to study textures in quark mass matrices. This type of theory relates quark masses to CKM matrix elements. I demonstrate how the recent precision results from the HPQCD and MILC collaborations for quark masses can be used to test some of these ideas.

Keywords:

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1. Introduction

The existence of different families of quarks and leptons is a puzzle. It is important to try to explain why the CKM matrix is diagonally dominant and to understand the values of the masses of the quarks. One way to understand the additional flavours is to look for some symmetry between the families [1, 2, 3, 4]. In the quark sector the way that the symmetries are searched for is to look for connections between the quark masses and the CKM matrix elements. There has been a long history of looking for patterns in the quark masses and the CKM matrix elements [5, 6].

Given that there is no theory that predicts symmetries between families, the subject is driven by the size of the errors on the CKM matrix elements and quark masses. The reduced errors bars on the CKM matrix from the experimental results from B factories [7] and improved theoretical predictions from techniques such as lattice QCD have already ruled out some proposed relationships between quark masses and CKM matrix elements. The PDG quote the current error on the strange quark mass as around 30%. However, this is accurate enough to rule out one of the original predictions: $V_{cb} \sim \frac{m_s}{m_b}$ of a 6 texture model [8]. The error on the strange quark mass should be

compared to the largest error on a CKM matrix element or ratio of CKM matrix, used in this paper, of 10%. The subject of mass and flavor mixing has been reviewed by Fritzsch and Xing [9], Froggatt [10], and Babu [11].

Recently the HPQCD collaborations have determined the masses of the strange, charm, and bottom quarks with an error of under 2% from unquenched lattice QCD [12, 13, 14]. The MILC collaboration had previously determined the ratio and sum of the masses of the up and down quarks [15]. The unprecedented precision stems from fitting lattice correlators in the continuum limit to continuum perturbation theory that depends on the masses of the charm and bottom quarks. The powerful techniques of multiloop QCD in the continuum have been used to compute the correlators to 4 loop order, hence this reduces one of the major systematic errors in lattice QCD calculations of quark masses, the conversion from the lattice results to the $\overline{MS}$ scheme. The light and strange quark masses are determined using the continuum limit of the ratios of quark masses. The suggestion to use the ratio of the light quark masses to the charm mass was originally suggested in the famous review article by Gasser and Leutwyler [16]. The basis of these results are generation of gauge configurations with 2+1 flavours of sea quark with light pion masses and multiple lattice spacings by the MILC collaboration [17, 18, 19]. Particularly important are the new gauge configurations at lattice spacings of 0.06 and 0.045 fm, generated by the MILC collaboration [19], that were crucial to taking a continuum limit for calculations with valence charm and bottom quarks.

The lattice QCD calculations by the HPQCD collaboration have been tested by prediction of the mass of the B_c meson [20] and the η_b meson [21]. A summary of the mass spectrum of heavy-heavy and heavy-light mesons is in [22]. The decay constants of the pion and kaon have been accurately computed [23]. The value of α_s extracted from the HPQCD collaboration [24, 12] is consistent with other non-lattice determinations in [25]. The staggered fermion formalism potentially has a problem with the technical issue of “rooting of determinants”. However, no theoretical work has found any problems [26, 27]. So for the arguments given in this paragraph the use of HPQCD’s quark masses is reasonable, rather than using the more conservative errors on the quark masses quoted by the PDG. Scholz [28] and Leutwyler [29] have recently reviewed the status of lattice QCD calculations of quark masses.

Apart from one simple example in section 4, I don’t discuss the relations between the quark masses and lepton masses predicted by some grand unified

theories [30]. This would be an important, but separate study and also require running the masses to the GUT scale [31] in a model dependent way. See [32, 33] for two recent studies.

In this paper I use the new precision results for quark masses from the HPQCD and MILC collaborations [14, 13, 15] to test some of the relations between CKM matrix elements and quark masses proposed by Chkareuli and Froggatt [34], and Fritzsch and Xing [35]. A goal of this project was to find out how accurately do we need to know the masses of the up, down and strange quarks.

2. An introduction to textures

The part of the standard model Lagrangian that describes the quark masses is

$$\mathcal{L} = -\overline{u}_L^i (M_u)_{ij} u_R^j - \overline{d}_L^i (M_d)_{ij} d_R^j + \text{h. c.} \tag{1}$$

where j is a index over flavour.

The mass matrices M_u and M_d are diagonalized to obtain the quark masses,

$$V_{uL} M_u V_{uR}^\dagger = diag(m_u, m_c, m_t) \tag{2}$$

$$V_{dL} M_d V_{dR}^\dagger = diag(m_d, m_s, m_b) \tag{3}$$

by the order 3 unitary matrices V_{uL}, V_{uR}, V_{dL}, and V_{dR}.

The CKM matrix is

$$U_{CKM} = V_{uL} V_{dL}^\dagger \tag{4}$$

A key prediction of the standard model is that the U_{CKM} matrix is unitary. The experimental program at the B factories have not found any significant deviation from unitarity [7]. Although there are perhaps some hints [36, 37]. The CKM matrix can be determined using the results from lattice QCD [36].

The idea is to look for some special structure in the mass mixing matrices M_u and M_d. For example there could be zeros in the mass matrices, which are known as textures [9]. One concern about this is that a special basis is chosen and the patterns could be removed by a transformation [38, 39]. The hope is that in some basis the physics is more transparent.

In table 1, I list the quark masses used in this study. I use the quark masses from the HPQCD and MILC collaborations and the ranges from the PDG [40]. The m_l mass is the average of quark masses of the up and down

Quantity	HPQCD/MILC	PDG
$m_u(2\text{ GeV})$	2.01 ± 0.10	2.55 ± 1.05
$m_d(2\text{ GeV})$	4.77 ± 0.15	5.04 ± 1.54
$m_s(2\text{ GeV})$	92.2 ± 1.3	105 ± 35
$m_c(m_c)$ GeV	1.273 ± 0.006	1.27 ± 0.11
$m_b(m_b)$ GeV	4.164 ± 0.023	4.20 ± 0.17
$m_t(m_t)$ GeV	-	160 ± 3
m_c/m_s	11.85 ± 0.16	-
m_u/m_d	0.42 ± 0.04	0.35 - 0.6
m_s/m_l	27.2 ± 0.03	25 - 30
m_b/m_c	4.51 ± 0.04	-

Table 1: Summary of quark masses and ratios of quark masses from the HPQCD and MILC collaborations [12, 13, 14, 15], the PDG [40], and Langenfeld et al. [41]

$V_{us} = 0.2255 \pm 0.0019$	$V_{ub} = (3.93 \pm 0.36)10^{-3}$	$V_{cb} = (41.2 \pm 1.1)10^{-3}$

Table 2: Values of the CKM matrix taken from the PDG [40]

quarks. I use the value for the top quark mass, in the $\overline{MS}$ scheme, from the work of Langenfeld et al. [41]. Many of the results from the HPQCD and MILC collaborations are ratio of quark masses, so where possible I use products of ratios of quark masses to compute the appropriate combination of quark masses. Otherwise I use the RunDec package [42] (using perturbative results from from [43, 44]) to run the quark masses to 2 GeV. I use the value of $\alpha_s^{\overline{MS}}(M_Z, n_f = 5) = 0.1184(6)$ from the HPQCD collaboration [24].

The CKM matrix are related to Yukawa couplings so must be renormalised [45]. I don't include any running of the CKM matrix [46], because the effects are thought to be small at low energies [9].

3. Results

In this section I will investigate the model for CKM parameters and quark masses introduced by Chkareuli and Froggatt [34]. They proposed the following ansatz for the up and down mass matrices.

$$M_u = \begin{pmatrix} 0 & 0 & \sqrt{m_u m_t} e^{ic_U} \\ 0 & m_c & 0 \\ \sqrt{m_u m_t} e^{-ic_U} & 0 & m_t - m_u \end{pmatrix} \quad (5)$$

$$M_d = \left(\begin{array}{ccc} 0 & \sqrt{m_d m_s} e^{i a_D} & 0 \\ \sqrt{m_d m_s} e^{-i a_D} & m_s & \sqrt{m_d m_b} e^{i b_D} \\ 0 & \sqrt{m_d m_b} e^{-i b_D} & m_b - m_d \end{array} \right) \quad (6)$$

where c_U, a_D, b_D are phases. The ansatz is based on an idea called Lightest Flavor Mixing. See the original paper for the motivation for the mass matrices [34].

Their model predicts the following relationships between the quark masses and CKM matrix elements.

$$V_{us} \sim \sqrt{\frac{m_d}{m_s}} = \sqrt{\frac{2}{1+\frac{m_u}{m_d}} \frac{m_l}{m_s}} \quad (7)$$

$$V_{cb} \sim \sqrt{\frac{m_d}{m_b}} = \sqrt{\frac{m_d}{m_s} \frac{m_s}{m_c} \frac{m_c}{m_b}} \quad (8)$$

$$V_{ub} \sim \sqrt{\frac{m_u}{m_t}} \quad (9)$$

In figure 1, I plot tests of the relationships in equations 7, 8 and 9 of Chkareuli and Froggatt [34]. The plots shows that the effect of the reduced errors on the quark masses from the HPQCD and MILC collaborations, over the PDG values. Figure 1 shows that the prediction for V_{cb} disagrees with the result from quark mass prediction of the HPQCD collaboration at the 10σ level.

Chkareuli and Froggatt [34] have another ansatz for the quark mass matrices, that predicts the relationship in equation 10.

$$\frac{\mid V_{ub} \mid}{\mid V_{cb} \mid} \sim \sqrt{\frac{m_u}{m_c}} \quad (10)$$

The quark masses from the PDG give $\sqrt{\frac{m_u}{m_c}} = 0.049(10)$, this disagrees with $\frac{|V_{ub}|}{|V_{cb}|}$=0.095(9) at the 4σ level.

Fritzsch and Xing [35] investigated a number of 4 texture mass matrices. A subset of the relations between quark masses and CKM matrix elements, that they derived, are below.

$$\frac{\mid V_{ub} \mid}{\mid V_{cb} \mid} \sim \sqrt{2\frac{m_u}{m_c}} \quad (11)$$

$$\mid V_{us} \mid \sim \sqrt{m_u/m_c + m_d/m_s} \quad (12)$$

$$\frac{\mid V_{td} \mid}{\mid V_{ts} \mid} \sim \sqrt{\frac{m_d}{m_s}} \quad (13)$$

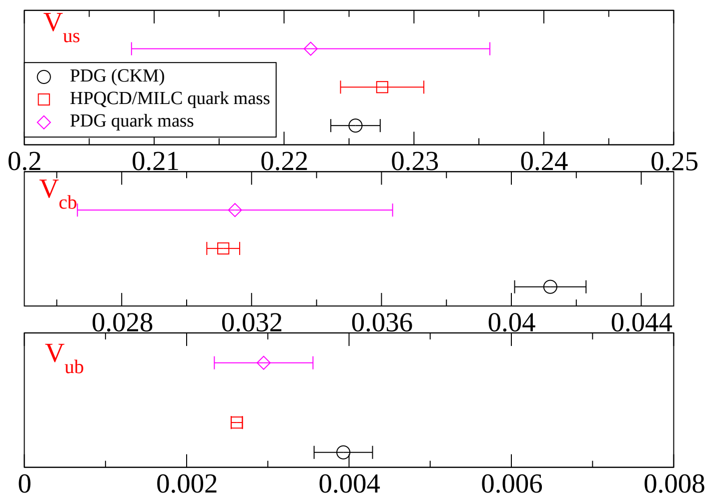

Figure 1: Test of the relationships 7, 8, and 9, between masses and CKM matrix elements predicted by Chkareuli and Froggatt [34]. The circles are the results for the CKM matrix elements. The squares and diamonds are the predictions for the CKM matrix elements in terms of quarks masses from the HPQCD/MILC collaborations and the PDG respectively.

CKM		Quark mass	
Element	Value	PDG	HPQCD/MILC
$\frac{\mid V_{ub}\mid}{\mid V_{cb}\mid}$	0.095(9)	0.069(15)	0.061(2)
$\mid V_{us}\mid$	0.2255(19)	0.22(4)	0.232(3)
$\frac{\mid V_{td}\mid}{\mid V_{ts}\mid}$	0.209(6)	0.22(1)	0.228(3)

Table 3: Comparison of CKM matrix elements from experiment with estimates from equation 11, 12 and 13, using the quark masses from the PDG and those from the HPQCD and MILC collaborations.

The numerical comparison of the relations between CKM matrix elements and quark masses derived by Fritzsch and Xing [35] are in table 3. The improved prediction for $\frac{|V_{ub}|}{|V_{cb}|}$ in equation 11 now only disagrees with the prediction of the quark masses from the PDG at the 1.6σ level. However, using the more accurate quark masses from the HPQCD/MILC collaboration, the predictions of Fritzsch and Xing [35] for $\frac{|V_{ub}|}{|V_{cb}|}$ and $\frac{|V_{td}|}{|V_{ts}|}$ disagree with experiment by over 3σ.

4. Conclusions

In this paper I have used the new high precision results for quark masses from the HPQCD and MILC collaborations to test some predictions for relations between CKM matrix elements. To illustrate the point I used some older ansatze [34, 35] for the quark matrices. The new accurate results for the quark masses from HPQCD and MILC essentially ruled out the two models. Many other studies of textures, such as [3, 47, 48], would benefit from more accurate values for the quark masses.

At the moment the smallest errors on the quark masses come from one unquenched lattice QCD calculation. Although in the introduction I briefly reviewed how well validated HPQCD's calculation were against experiment, it would clearly be preferable to have accurate results from other lattice formalisms. Recent reviews of dynamical lattice QCD calculations show that many groups have access to data with pion masses at least at 300 MeV, multiple lattice spacings and volumes [49, 50], so the prospects for reductions in the errors of quark masses from other formulations is good.

It would be interesting to also use precision lattice QCD results for quark masses to test the predictions of grand unified theories. In GUTs there could be relationships between lepton and quark masses at some renormalization

scale. For example Georgi and Jarlskog suggested [30] (see Babu [11] for a review), a relationship between the masses of the down and strange quarks, and the masses of the muon and electron masses.

$$\frac{m_s}{m_d} = \frac{1}{9}\frac{m_\mu}{m_e} \tag{14}$$

The factor of $\frac{1}{9}$ in equation 14 is from the square of the number of colours. Experimentally, $\frac{1}{9}\frac{m_\mu}{m_e} = 21.3$. The new lattice QCD numbers from the HPQCD collaboration, give $\frac{m_s}{m_d} = 19.3(5)$, while the current ranges from the PDG give $\frac{m_s}{m_d} = 20.3(2.5)$. So the new results from HPQCD for the quark masses are inconsistent with the relation in equation 14 at the 4σ level at low energy. However, equation 14 only needs to hold at the unification scale and a more detailed study would include the effects of additional particles, such as those from SUSY models [32, 33].

To test the texture relations the errors on the ratios of quark masses need to be at least the relative size of the errors on the relevant CKM matrix elements. For example the V_{us} CKM matrix element is currently known to under 1% accuracy, so a similar accuracy is required for ratios of quark masses for a good test of the predictions of textures. The results from improved lattice QCD calculations will also reduce the errors on CKM matrix elements, such as V_{us} [51].

The predictions in equations 7, 8, 9 and 14 critically depend on the values of the masses of the up and down quarks. Currently the lattice results for the masses of the up and down quarks are based on the method developed by the MILC collaborations [15]. In particular the majority of the error on the ratio m_u/m_d is due their treatment of electromagnetic effects. There are new lattice QCD calculations [52, 53] that explicitly include electromagnetism [54, 55, 52], or estimate the corrections [56, 57] that should reduce the errors on the masses of the up and down quarks obtained from lattice QCD calculations.

As reviewed by Babu [11], the structure of the quark mass matrices could be caused by "Flavon fields" [1]. Although the dynamics that generates the quark mass matrices could be at the Planck energy, there could also be measurable effects at the LHC [58, 59, 60].

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